The Pythagorean Theorem

Vocabulary!

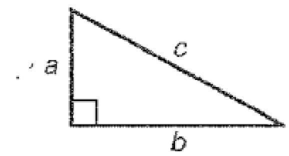
Pythagorean triple – A set of three positive integers *a, b,* and *c,* that satisfy the equation $c^2 = a^2 + b^2$

3, 4, 5   8, 15, 17
5, 12, 13   7, 24, 25

THEOREM 9.4: PYTHAGOREAN THEOREM

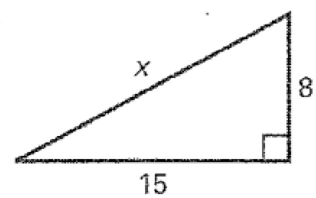
In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

$c^2 = a^2 + b^2$



Example 1: ***Finding the Length of a Hypotenuse***

Find the length of the hypotenuse of the right triangle. Tell whether the side lengths form a Pythagorean triple.



Solution

$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$ **Pythagorean Theorem**

$x^2 = 15^2 + 8^2$

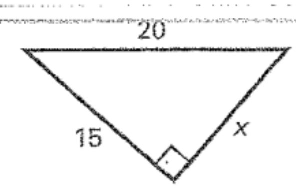
$= 225 + 64$

$\sqrt{x^2} = \sqrt{289} \rightarrow x = 17$

$\boxed{8, 15, 17}$

Example 2: ***Finding the Length of a Leg***

Find the length of the leg of the right triangle.



Solution

$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$ **Pythagorean Theorem**

$20^2 = 15^2 + x^2$

$400 = 225 + x^2$

$\sqrt{175} = \sqrt{x^2}$

$\sqrt{175} = \sqrt{25} \cdot \sqrt{7} = 5\sqrt{7}$

175 → 25, 7

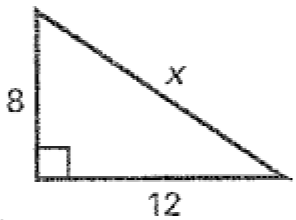
25 → 5, 5

$= 5\sqrt{7}$

Find the value of ***x***. **Simplify answers that are radicals.** **Then tell whether the side lengths form a Pythagorean triple.**

$\boxed{7, 24, 25}$

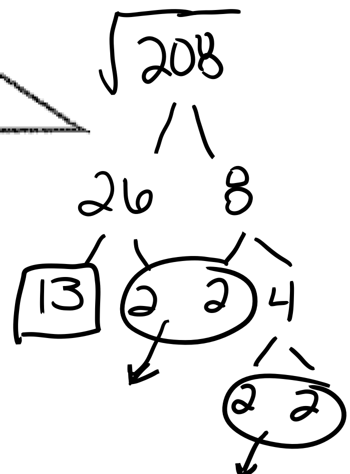
1.



$8^2 + 12^2 = x^2$

$64 + 144 = x^2$

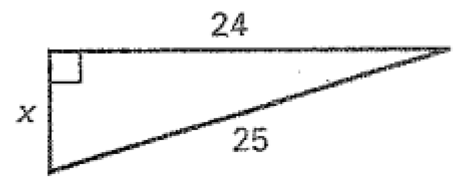
$\sqrt{208} = \sqrt{x^2}$



$2 \cdot 2 \cdot \sqrt{13} = 4\sqrt{13}$

not pyth triple

2.



$x^2 + 24^2 = 25^2$

$x^2 + 576 = 625$

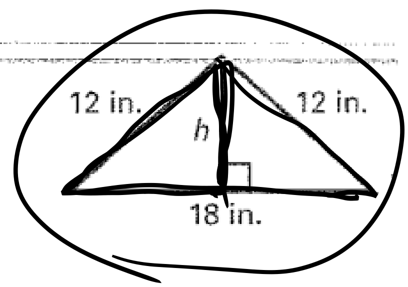
$\sqrt{x^2} = \sqrt{49}$

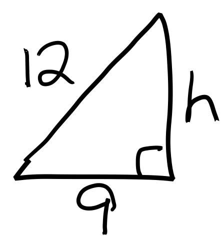
$x = 7$

Yes → 7, 24, 25

Example 3: ***Finding the Area of a Triangle***

Find the area of the triangle to the nearest tenth of a square inch.





Solution

$12^2 = 9^2 + h^2$ Pythagorean Theorem

$144 = 81 + h^2$

$63 = h^2$

$\sqrt{63} = \sqrt{9}\,\sqrt{7} = 3\sqrt{7}$

$A = \frac{1}{2}bh$ Area of a triangle

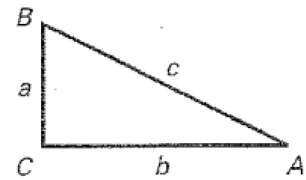
$A = \frac{1}{2}(18)(3\sqrt{7})$

$= 71.4\ in^2$

The Converse of the Pythagorean Theorem

THEOREM 9.5: CONVERSE OF THE PYTHAGOREAN THEOREM

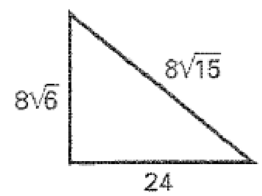
If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.



If $c^2 = a^2 + b^2$, then ΔABC is a right triangle.

Example 1: ***Verifying Right Triangles***

Tell whether the triangle at the right is a right triangle.



Solution

Let c represent the length of the longest side of the triangle. Check to see whether the side lengths satisfy the equation $c^2 = a^2 + b^2$.

$$(8\sqrt{15})^2 \stackrel{?}{=} (8\sqrt{6})^2 + 24^2$$

$$8^2 \cdot (\sqrt{15})^2 \stackrel{?}{=} 8^2 \cdot (\sqrt{6})^2 + 24^2$$

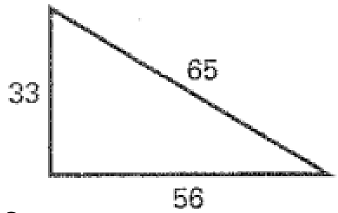
$$64 \cdot 15 \stackrel{?}{=} 64 \cdot 6 + 576$$

$$960 \stackrel{?}{=} 384 + 576$$

$$960 = 960 \longrightarrow \text{right } \Delta$$

Tell whether the triangle is a right triangle.

1.



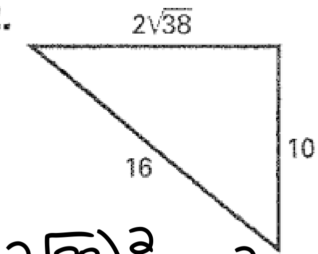
$65^2 = 33^2 + 56^2$

$4225 = 1089 + 3136$

$4225 = 4225$

$\hookrightarrow$ right Δ

2.



not right Δ

$16^2 = (2\sqrt{38})^2 + 10^2$

$16^2 = 2^2 \cdot (\sqrt{38})^2 + 10^2$

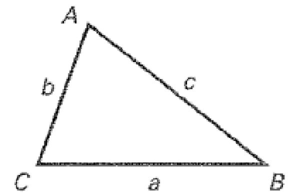
$256 = 4 \cdot 38 + 100$

$256 = 152 + 100 \rightarrow 256 \neq 252$

THEOREM 9.6

If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, then the triangle is acute

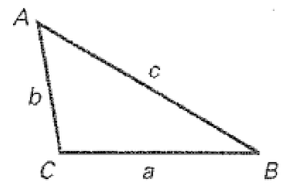
If $c^2 < a^2 + b^2$, then $\triangle ABC$ is acute



THEOREM 9.7

If the square of the length of the longest side of a triangle is greater than the sum of the squares of the lengths of the other two sides, then the triangle is obtuse.

If $c^2 > a^2 + b^2$, then $\triangle ABC$ is obtu .



Example 2: *Classifying Triangles*

Decide whether the set of numbers can represent the side lengths of a triangle. If they can, classify the triangle as *right, acute, or obtuse.*

a. 28, 40, 48

b. 5.7, 12.2, 13.9

Solution

Compare the square of the length of the longest side with the sum of the squares of the lengths of the two shorter sides.

$48^2 \ ? \ 28^2 + 40^2$

$2304 \quad 784 + 1600$

$2304 < 2384$

acute

$13.9 \ ? \ 5.7^2 + 12.2^2$

$193.21 \quad 32.49 + 148.84$

$193.21 > 181.33$

obtuse

Can the numbers represent the side lengths of a triangle? If so, classify

3. 16, 30, 34

right

4. 8, 13, 22

not a $\triangle$

5. 6, 9, 12

obtuse